MISCELLANEOUS QUESTIONS.

EDITED BY R. D. CARMICHAEL.

QUESTION.

6. In what ways and to what extent will the teaching of mathematics and the content of the curriculum probably be affected by the increasing demand for vocational training?

REPLIES.

2. It is clear, on the most casual observation, that the average college curriculum in mathematics is unbalanced in two respects: (1) algebra predominates over geometry, (2) analytic geometry predominates over our synthetic geometry. This reacts in two undesirable ways: (1) to deprive the college student of a rich and interesting field of study, (2) to give a very one-sided training for prospective teachers of high school mathematics. What can be done to remedy this situation?

IV. Remarks by R. D. Carmichael, Indiana University.

The discussion of this question both by Professor Bussey in the issue of November, 1913, and by several correspondents in the January issue has been stimulating. There can be no doubt that our college curriculum is unbalanced in some important respects. In fact it would probably be hard to give a good reason why certain topics are retained in our courses and still harder to say why others have not been introduced.

It seems clear that the claim of synthetic geometry is a valid one. The beauty of the subject cannot be doubted, and its appeal to the majority of students will probably be granted. It adds to the esthetic delight of one's intellectual life when he is able to picture space as shot through with the beautiful relations which have been created by geometers. It gives him a language of precision and imagery in which to set forth the relations of phenomena in the world of external things through which he must get about in his daily experience.

The purpose of these remarks, however, is not primarily to emphasize these matters, although the present state of our curriculum shows that they are in need of emphasis; it is rather to point out what appears to be a wrong judgment concerning analysis or a wrong emphasis of its difficulties, as expressed in the following words in one of the communications referred to: "There comes a time in the life of every mathematical student when the continual grind of analysis becomes unbearable."

To the present writer analysis is not a "grind" and has never been; on the other hand, it throws over him a peculiar fascination. His deepest intellectual pleasure is the experience of beholding the beautiful threads of connection among these abstract creations of the mind and in witnessing the marvelous and unexpected ways in which they have contributed to our better understanding of the external world and its phenomena.

If this experience were confined to a single individual it should pass unmentioned. But there are not a few who feel the same sense of charm in the

presence of the far-reaching results of analysis, and find in them (directly or indirectly) the chief source of their pleasure in mathematical discipline; and this experience has been theirs through the whole course of their study.

It is doubtless true, however, that many students find analysis anything but pleasant and that this would continue to be their experience although they gave much attention to its cultivation. For these we need synthetic geometry in all its beauty; and not for these only, but for all students who come to us for mathematical training. Let us adjust our courses so that every college student may have the opportunity to go into this rich and interesting field of study.

3. In connection with the theory of the conduction of electricity through gases, one is led to the differential equation

$$y\frac{d^2y}{dx^2} + a\left(\frac{dy}{dx}\right)^2 + b\frac{dy}{dx} = cy + d = 0, \tag{1}$$

where a, b, c, d are constants. For unrestricted values of a, b, c, d the solution of this differential equation presents peculiar difficulties, the series solutions obtained by the customary methods having (apparently) too small a range of convergence to be satisfactory from the point of view of electrical theory. The general solution of this equation is wanted in case it can be found. If no general solution is obtained for unrestricted a, b, c, d, it is desirable to know special values of a, b, c, d or special relations among a, b, c, d which make it possible to find the general solution; and this solution is desired in each case.

I. Remarks by X.

A list of some cases in which the differential equation (1) may be solved by elementary means is the following:

1. When $b = c = d = 0$, $a \neq 0, -1$. The general solution in this case is readily found to be

$$y = (\alpha x + \beta)^{\frac{1}{a+1}},$$

where α and β are arbitrary constants.

2. When $b = c = d = 0$, $a = -1$. The general solution is

$$y = e^{\alpha x+\beta},$$

where α and β are arbitrary constants.

3. When $a = c = d = 0$, $b \neq 0$. It is easy to show that the general solution is given by the equation

$$\int_0^{\alpha y} \frac{dt}{\log t} + bx + \beta = 0,$$

where α and β are arbitrary constants.

4. When $a = b = d = 0$. The general solution is

$$y = -\frac{cx^2}{2} + \alpha x + \beta.$$

5. When $a = b = c = 0$, $d \neq 0$. The general solution is given by the equat on

$$\int_0^y \frac{dt}{\sqrt{\alpha - 2d \log t}} + x + \beta = 0.$$

6. When $c = d = 0$, $a \neq 0$, $b \neq 0$. The general solution is given by the equation

$$\int_0^y \frac{at^a dt}{bt^a - \alpha} + x + \beta = 0.$$

7. When $a = b = 0$, $c \neq 0$, $d \neq 0$. The general solution is given by the equation

$$\int_0^y \frac{dt}{\sqrt{\alpha - 2ct - 2d \log t}} + x + \beta = 0.$$

Returning now to the general equation (1), put

$$t = \alpha x + \beta,$$

where α and β are any constants except that α is to be different from zero. Then the differential equation becomes

$$y \frac{d^2y}{dt^2} + a\left(\frac{dy}{dt}\right)^2 + \frac{b}{\alpha}\frac{dy}{dt} + \frac{c}{\alpha^2}y + \frac{d}{\alpha^2} = 0. \tag{2}$$

If the original equation (1) has the solution

$$y = f(x, a, b, c, d), \tag{3}$$

then equation (2) has the solution

$$y = f\left(t, a, \frac{b}{\alpha}, \frac{c}{\alpha^2}, \frac{d}{\alpha^2}\right),$$

and hence the original equation (1) has the solution

$$y = f\left(\alpha x + \beta, a, \frac{b}{\alpha}, \frac{c}{\alpha^2}, \frac{d}{\alpha^2}\right). \tag{4}$$

Therefore, *if any solution* (3) *of* (1) *is known, actually involving x, a solution exists in the form* (4) *involving two arbitrary constants.* This fact may be of use in obtaining the general solution of (1).

NOTES AND NEWS.

UNDER THE DIRECTION OF FLORIAN CAJORI.

Professor M. Fréchet, of Poitiers, France, is expected to lecture at the University of Illinois during the next academic year.